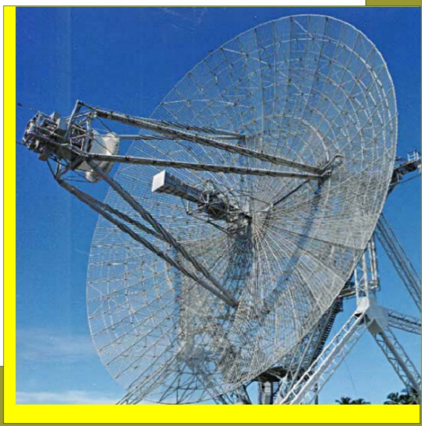

# GATE / PSUs

## ELECTRONICS ENGINEERING-ECE

STUDY MATERIAL

# SIGNAL & SYSTEM

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**STUDY MATERIAL**

# SIGNAL & SYSTEM

# CONTENTS

# CHAPTER-1
# SIGNALS AND SYSTEM

## 1.1 INTRODUCTION

The concept and theory of signals and systems are needed in almost all electrical engineering fields and in many other engineering and scientific disciplines as well. In this chapter we introduce the mathematical description and representation of signals and systems and their classifications. We also define several important basic signals essential to our studies.

**What are Signals?**

Signals are represented as functions of one or more independent variables. All signals carry some kind of information.

**Examples:** Voice signal $\rightarrow$ 300 Hz to 3400 Hz

Audio signal $\rightarrow$ 20 Hz to 20 kHz

## 1.2 Classification of signals

Signals can be classified on the basis of different parameters:

**A. Continuous time and discrete time signals**

A signal is said to be continuous time signal if it is defined at all values of time parameter *'t'* or we can say '*t*' must be continuous variable.

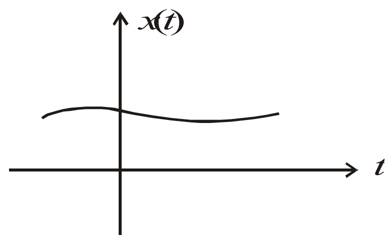



**Figure: Continuous time signal**

A discrete-time signal can be obtained from sampling a continuous time signal $x(t)$ at discrete time instants like $t$ = 0, 1, 2, ……. It is represented by $x[n]$. Various values are given by $x[0]$, $x[1]$ ……..Here$x[0]$, $x[1]$ …… are called samples of the continuous time signal $x(t)$.

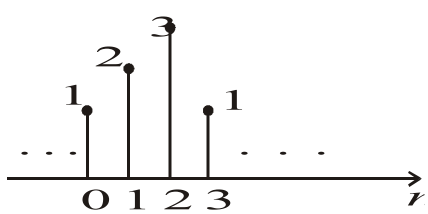



**Figure: Discrete-time signal**

**Note:** Sampling : $t = nT_s$

$T_s \rightarrow$ Sampling interval

In signals analysis generally $T_s = 1$

$\Rightarrow \quad t = n$

**B. Analog and Digital Signals**

If a continuous-time signal $x(t)$ can take on any value in the continuous interval $(a, b)$, where a may be $-\infty$ and $b$ may be $+\infty$, then the continuous-time signal $x(t)$ is called an analog signal. If a discrete-time signal $x[n]$ can take on only a finite number of distinct values, then we call this signal a digital signal.Digital signals are discretized in both time and value.

**C. Real and Complex Signals**

A signal $x(t)$ is a real signal if its value is a real number, and a signal $x(t)$ is a complex signal if its value is a complex number. A general complex signal $x(t)$ is a function of the form

$$x(t) = x_1(t) + jx_2(t) \quad \text{.... (i)}$$

Where $x_1(t)$ and $x_2(t)$ are real signal and $j = \sqrt{-1}$.

Note that in Eq. (i) $t$ represent either a continuous or a discrete variable.

Advantage of complex signal: Number of operations required to perform same task is get reduced.

**D. Deterministic and Random Signals**

*Deterministic* signals are those signals whose values are completely specified for any given time. Thus, a deterministic signal can be modeled by a known function of time $t$. Random signals are those signals that take random values at any given time and must be characterized statistically. Random signals will not be discussed in this text.

For example: $y(t) = 2t + 1$

So $y(t)$ is defined and deterministic for each value of $t$.

And suppose temperature of a city is a random signal which can take arbitrary values.

**E. Even and Odd Signals**

A signal $x(t)$ or $x[n]$ is referred to as an even signal if

$$x(-t) = x(t)$$
$$x[-n] = x[n] \quad \text{...(i)}$$

A signal $x(t)$ or $x[n]$ is referred to as an odd signal if

$$x(-t) = -x(t)$$
$$x[-n] = -x[n] \quad \text{...(ii)}$$

Any signal $x(t)$ or $x[n]$ can be expressed as a sum of two signals, one of which is even and one of which is odd. That is,

$$x(t) = x_e(t) + x_o(t) \quad \text{...(iii)}$$
$$x[n] = x_e[n] + x_o[n]$$

where $x_e(t) = \frac{1}{2}\{x(t) + x(-t)\}$ even part of $x(t)$

$$x_e[n] = \frac{1}{2}\{x[n] + x[-n]\} \text{ even part of } x[n] \quad \text{...(iv)}$$

$$x_0(t) = \frac{1}{2}\{x(t) - x(-t)\} \text{ odd part of } x(t)$$

$$x_o[n] = \frac{1}{2}\{x[n] - x[-n]\} \text{ odd part of } x[n] \quad \text{...(v)}$$

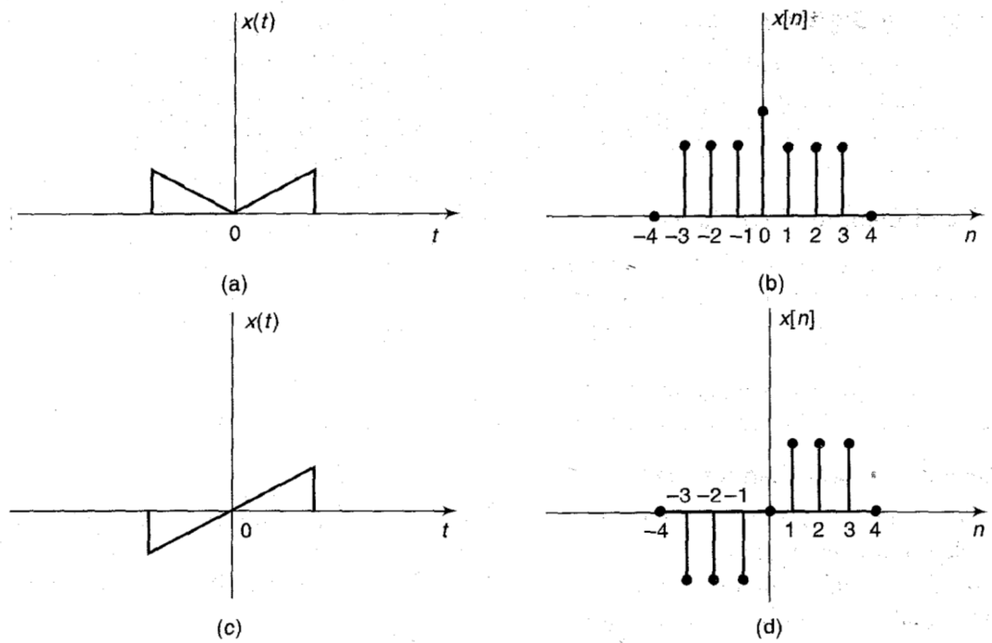



**Figure:** Example of even signals (a and b) and odd signals (c and d)

So, from the above formula, we can split any signal into its even and odd parts. And subsequently we can find the even and odd part of any signal $x(t)$**.**

**Properties of even and odd signals:**

(i) Even signals are symmetrical about y axis.

(ii) Odd signals are symmetrical about origin.

(iii) Addition of two even signals are even and addition of two odd signals are always odd.

(iv) Multiplication of two even signals and two odd signals are always even.

(v) Multiplication of even and odd signals are odd.

(vi) Area of odd signal = 0

$$\int_{-\infty}^{\infty} x_0(t)dt = 0 \qquad \& \qquad \sum_{n=-\infty}^{\infty} x_0(n) = 0$$

(vii) $$\int_{-\infty}^{\infty} x_e(t)dt = 2\int_{0}^{\infty} x_e(t)dt$$

Where $x_e(t) \rightarrow \textit{Even signal}$.

Even & odd for complex signals:

A signal x(t) or x(n) is referred to as an even conjugate if

$$x(t) = x^*(-t)$$
$$x(n) = x^*(-n) \quad \ldots\text{(vi)}$$

**Example:**

$$x(t) = e^{Jt}$$
$$x^*(t) = e^{-Jt}$$
$$x^*(-t) = e^{Jt} = x(t)$$

A signal x(t) or x(n) is referred to

as an odd conjugate if

$$x(t) = -x^*(-t)$$
$$x(n) = -x^*(-n) \quad \ldots\text{(vii)}$$

**Example:** $x(t) = Je^{Jt}$

$$x^*(t) = -Je^{-Jt}$$

$$x^*(-t) = -Je^{Jt} = -x(t)$$

**Note:** For even conjugate signal real part is always even & imaginary part is always odd.

Similarly for odd conjugate signal real part is always odd and imaginary part is always even.

Now any signal x(t) or x(n) can be expressed as a sum of two signals, one of which is even conjugate & one of which is odd conjugate , that is

$$x(t) = x_{ec}(t) + x_{oc}(t)$$
$$x(n) = x_{ec}(n) + x_{oc}(n) \quad \ldots\text{(viii)}$$

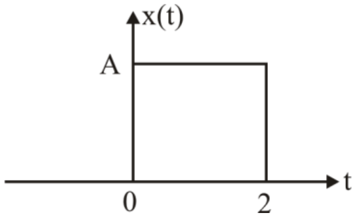
where $x_{ec}(t) = \frac{1}{2}\left[x(t) + x^*(-t)\right]$

$$x_{ec}(n) = \frac{1}{2}\left[x(n) + x^*(-n)\right] \quad \ldots\text{(ix)}$$

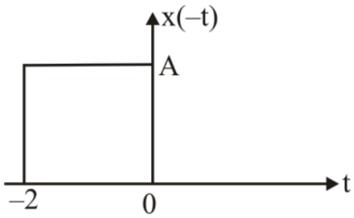
$$x_{oc}(t) = \frac{1}{2}\left[x(t) - x^*(-t)\right]$$

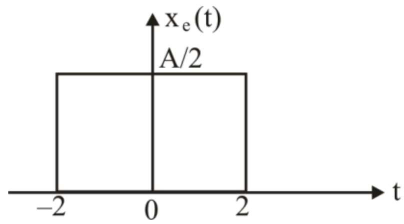
$$x_{oc}(n) = \frac{1}{2}\left[x(n) - x^*(-n)\right] \quad \ldots\text{(x)}$$

**Example:** Determine even and odd part of

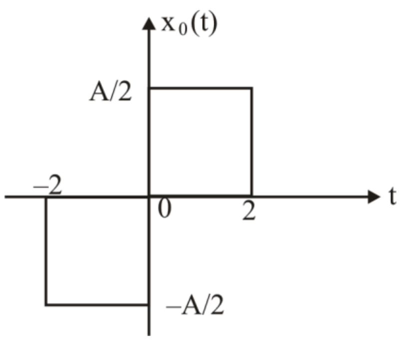



**Solution:**

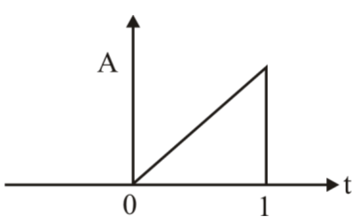



**&**




**Problem:** Determine even & odd part of




**Example:** If x(n) is given by

$$x(n) = \{\underset{\uparrow}{3+2J},\ 2-J,\ -1-3J,\ -2+2J\}$$

determine $x^*(-n)$

**Solution:**
$$x^*(n) = \begin{bmatrix} 3-2J, & 2+J, & -1+3J, & -2-2J \\ & \uparrow & & \end{bmatrix}$$

$$x^*(-n) = \begin{bmatrix} -2-2J, & -1+3J, & 2+J, & 3-2J \\ & & \uparrow & \end{bmatrix}$$

**Problem:**
$$x(n) = \begin{bmatrix} 4+2J, & 5-3J, & -1-2J, & -4+J \\ & \uparrow & & \end{bmatrix}$$

Determine the even conjugate part of x(n)

**F. Periodicand Non-periodic Signals**

A continuous-time signal $x(t)$ is said to be periodic with period T if there is a positive nonzerovalue of T for which

$x(t) = x(t+T)$ for all t ...(i)

$\Rightarrow x(t+mT) = x(t)$ ... (ii)

For all $t$ and any integer m. The fundamental period $T_0$ of $x(t)$ is the smallest positive value of $T$ forwhich Eq. (i) holds. Any continuous-time signal which is not periodic is called a non-periodic (or aperiodic).

**Procedure for finding the time period:**

(i) Determine the individual time period.

$T_1, T_2, T_3$ ...

(ii) Calculate $\frac{T_1}{T_2}, \frac{T_1}{T_3}$ ...

(iii) If ratios of step (ii) are rational number then the overall signal is periodic.

(iv) Calculate LCM of denominators of step (ii) & denoted as Z.

(v) Overall time period T = $ZT_1$

Periodic discrete-time signals are defined analogously. A sequence (discrete-time signal) $x[n]$ isperiodic with period $N$ if there is a positive integer $N$ for which

$x[n+N] = x[n]$ all n ...(iii)

$\Rightarrow x[n+mN] = x[n]$ ... (iv)

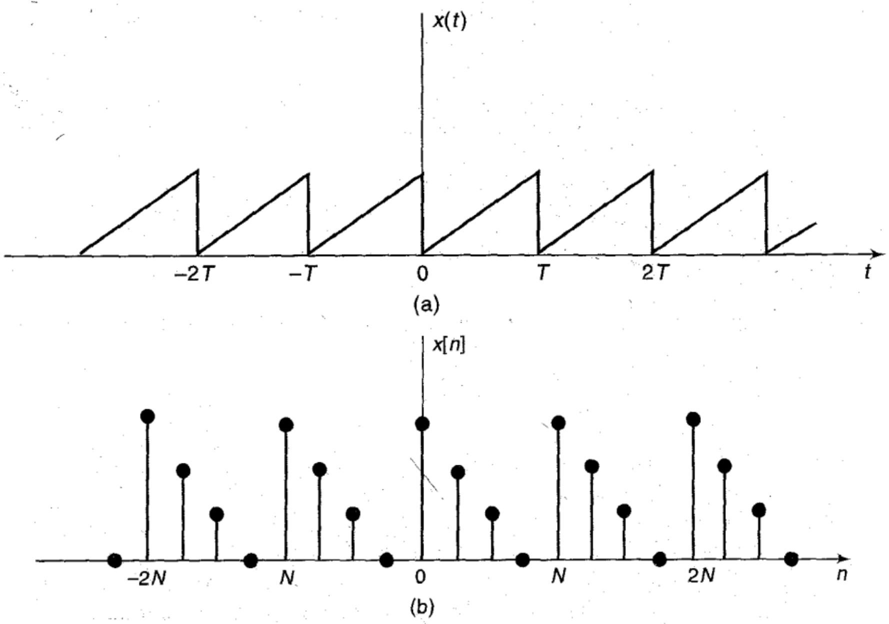



**Figure:** Examples of Periodic Signals

for all $n$ and any integer $m$. The fundamental period $N_0$ of $x[n]$ is the smallest positive integer $N$ for which Eq. (iii) holds. Any sequence which is not periodic is called a non-periodic (or aperiodic) sequence.

**Procedure for finding the time period: (Individual)**

$\frac{2\pi}{\omega} = \frac{N}{K}$, is minimum value of integer for which N is also integer.

**Note:** (*i*) Fundamental period of constant signal is undefined.

(*ii*) Sum of two continuous periodic signals may not be periodic but sum of two periodic sequences is always periodic.

**(iii)** Individual sine, cosine signals is continuous domain are always periodic.

**(iv)** Individual sine, cosine signals is discrete domain may not be periodic.

**(v)** One sided signals are never periodic.

**Example:** Determine the time period of x(t) if x(t) is periodic, where x(t) = sin 50 πt + cos 60 πt + cos 60 πt.

**Solution:** $\omega_1 = 50\pi, \quad \omega_2 = 60\pi$

$$T_1 = \frac{2\pi}{50\pi}, \qquad T_2 = \frac{2\pi}{60\pi}$$

$$T_1 = \frac{1}{25}, \qquad T_2 = \frac{1}{30}$$

$\frac{T_1}{T_2} = \frac{6}{5} = 1.2,$ rational number

$\Rightarrow \quad T = ZT_1$, where $Z = 5$

$\Rightarrow \quad T = 5 \times \frac{1}{25} = \frac{1}{5}\text{sec.}$

**Problem:** Determine the time period of $x(t)$; *if* $x(t)$ is periodic, where $x(t) = \cos\left(\frac{4\pi}{3}t\right).\sin\left(\frac{8\pi t}{5}\right)$

**Example**: Determine time period of $x(n)$ where $x(n) = \cos\left(\frac{7\pi}{3}n\right)$

**Solution:** $\omega = \frac{7\pi}{3}$

$$\frac{2\pi}{\omega} = \frac{2\pi \times 3}{7\pi} = \frac{N}{K}$$

$\Rightarrow \quad \frac{N}{K} = \frac{6}{7} \;\; So \;\; for \;\; K = 7,\, N = 6$

**Problem:** Check weather $u(n) + u(-n)$ is periodic or not, where u(n) is unit step sequence.

**G. Energy and Power Signals**

For an arbitrary continuous-time signal *x*(*t*), the normalized energy content *E* of *x*(*t*) is defined as

$$E = \int_{-\infty}^{\infty} \left|x(t)\right|^2 dt \qquad \ldots \text{(i)}$$

The normalized average power *P* of *x*(*t*) is defined as

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} \left|x(t)\right|^2 dt \qquad \ldots \text{(ii)}$$

Similarly, for a discrete-time signal *x*[*n*], the normalized energy content *E* of *x*[*n*] is defined as

$$E = \sum_{n=-\infty}^{\infty} \left|x[n]\right|^2 \qquad \ldots \text{(iii)}$$

The normalized average power P of x[n] is defined as

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left|x[n]\right|^2 \qquad \ldots \text{(iv)}$$

Based on definitions (i) to (iv), the following classes of signals are defined:

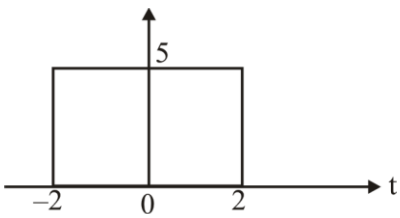
1. *x*(*t*) (or *x*[*n*]) is said to be an energy signal (or sequence) if and only if $0 < E < \infty$, and so $P = 0$.
2. *x*(*t*) (or *x*[*n*]) is said to be a power signal (or sequence) if and only if $0 < P < \infty$, thus implying that $E = \infty$.
3. Signals that satisfy neither property are referred to as neither energy signals nor powersignals.
4. Periodic signals are generally power signals.

**Properties of energy signals:**

If energy of x(t) is E then,

$$(i)\ \ Energy\left[x(at+b)\right]=\frac{E}{|a|}$$
$$(ii)\ Energy\left[Ax(t)\right]=A^2E$$
$$(iii)\ Energy\left[A+x(t)\right]=\infty$$

**Example:** check whether the signal x(t) is energy signal or power signal hence calculate energy or power.

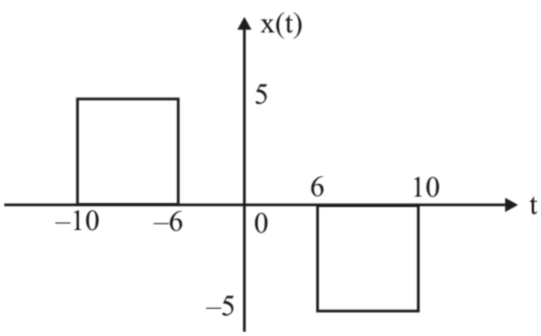



**Solution:** $Energy=\int\limits_{-\infty}^{\infty}x^2(t)dt$

$$=\int\limits_{-2}^{2}5^2\,dt=25\times 4=100J$$

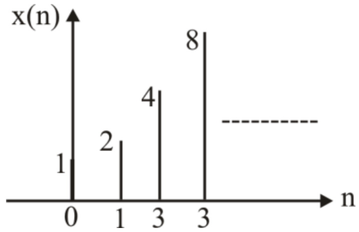
Energy is finite, hence energy signal.

**Problem:** check whether the signal x(t) is energy signal or power signal hence calculate energy or power.




**Example:** Check weather $x(n)=2^n u(n)$ is energy signal or power or neither nor.

**Solutions:** $x(n)=2^n u(n)$




It is increasing function n so at $n \to \infty, \ x(n) \to \infty$. So is neither energy nor power.

**Problem:** Check weather signal x(n) = u(n) is energy signal or power signal, hence calculate energy or power.

**Note:**

(i) Addition of energy signal and power signal is always power signal.

(ii) If for time $\to \pm\infty$, signal is also tends to $\infty$ then it is neither energy nor power signal.

(iii) If for time $\to \pm\infty$, signal value is zero then it is energy signal,

Causal and nor causal signal:

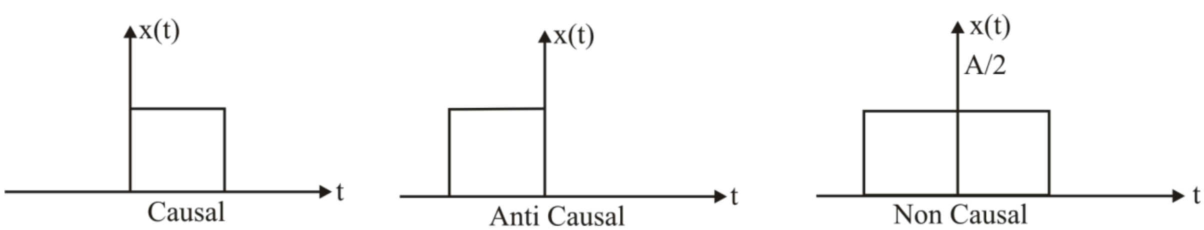
If $x(t) = 0 \ for \ t < 0$
or $x(n) = 0 \ for \ n < 0$

Then signal is causal signal

If $x(t) = 0 \ for \ t > 0$ or $x(n) = 0 \ for \ n > 0$ then signal is anti-causal.

If $x(t) \neq 0, \ t < 0 \ \& \ t > 0$ or $x(n) \neq 0, \ t > 0 \ \& \ t < 0$

Then signal is non causal



## 1.3 BASIC CONTINUOUS-TIME SIGNALS

**A. The unit step functions**

The unit step function u(t), also known as the Heaviside unit function, is defined as

$$u(t)=\begin{cases}1 & t>0\\ 0 & t<0\end{cases}$$

Note that it is discontinuous at t = 0 and that the value at t = 0 is undefined. Similarly, the shifted unit step function $u(t-t_0)$ is defined as

$$u(t-t_0)=\begin{cases}1 & t>t_0\\ 0 & t<t_0\end{cases}$$

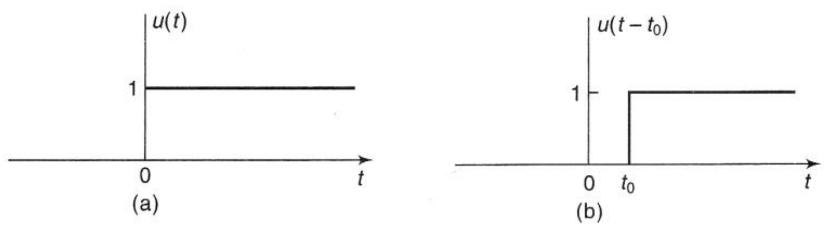



**Figure:** (a) Unit step function, (b) Shifted unit step function

**Gibb's Phenomenon:** If a signal is discontinuous at a point, then signal value at the point of discontinuity will be the average value of signal value taken just before and after at discontinuous point.

$$u(t=0)=\frac{u(0^+)+u(0^-)}{2}=\frac{0+1}{2}=\frac{1}{2}$$

**B. The unit impulse function**

The unit impulse function δ(*t*), also known as the Dirac delta function, plays a central role in system analysis.

$$\delta(t)=\begin{cases}0 & t\neq 0\\ \infty & t=0\end{cases}$$

$$\int_{-\infty}^{\infty}\delta(t)\,dt=1$$

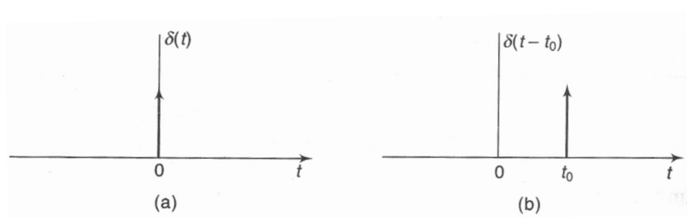



**Figure** (a) Unit Impulse Function; (b) Shifted Unit Impulse Function

**Properties of impulse function:**

(i) Sampling of integral property

$$\int_a^b \phi(t)\,\delta(t-t_0)\,dt=\begin{cases}\phi(t_0) & a<t_0<b\\ 0 & a<b<t_0 \;\; or \;\; t_0<a<b\\ undefined & a=t_0 \;\; or \;\; b=t_0\end{cases} \quad \ldots\text{(i)}$$

(ii) Scaling property:

$$\delta(at) = \frac{1}{|a|}\delta(t) \qquad \text{… (ii)}$$

(iii) Impulse is even function of time :

$$\delta(-t) = \delta(t) \qquad \text{… (iii)}$$

(iv) Product property:

If $x(t)$ is continuous at $t = t_0$.

$$x(t)\,.\,\delta(t - t_0) = x(t_0)\,.\,\delta(t - t_0) \qquad \text{… (iv)}$$

## C. Complex Exponential Signals:

The complex exponential signal

$$x(t) = e^{j\omega_0 t} \qquad \text{… (i)}$$

is an important example of a complex signal. Using Euler's formula, this signal can be defined as

$$x(t) = e^{j\omega_0 t} = \cos\omega_0 t + j\sin\omega_0 t \qquad \text{… (ii)}$$

Thus, $x(t)$ is a complex signal whose real part is $\cos\omega_0 t$ and imaginary part is $\sin\omega_0 t$. An important property of the complex exponential signal $x(t)$ is that it is periodic. The fundamental period $T_0$ of $x(t)$ is given by

$$T_0 = \frac{2\pi}{|\omega_0|} \qquad \text{… (iii)}$$

Note that $x(t)$ is periodic for any value of $\omega_0$

**General Complex Exponential Signals** Let $s = \sigma + j\omega$ be a complex number. We define $x(t)$ as

$$x(t) = e^{st} = e^{(\sigma + j\omega)t} = e^{\sigma t}(\cos\omega t + j\sin\omega t) \text{------------------(iv)}$$

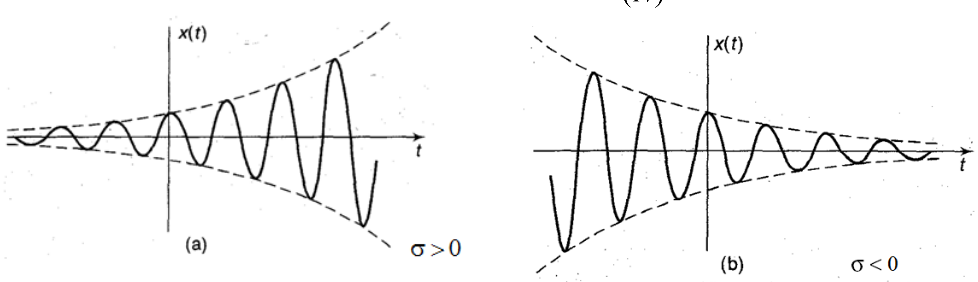
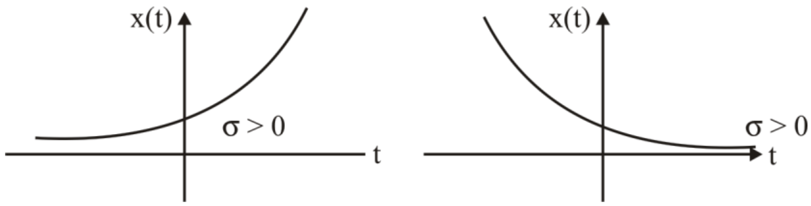



**Figure:** (a) Exponentially Increasing sinusoidal signal (b) Exponentially decreasing sinusoidal Signal

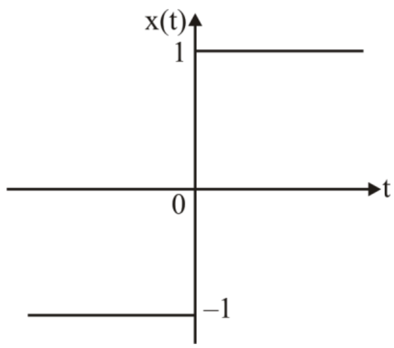
**Real exponential signals** Note that if $s = \sigma$ (a real number), then Eq. (iv) reduces to a real exponential signal

$$x(t) = e^{\sigma t} \qquad \dots \text{(v)}$$

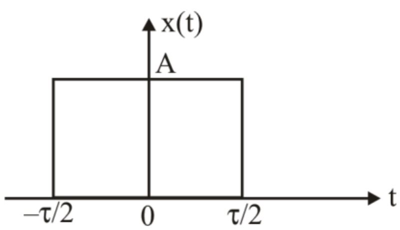
As illustrated in Figure below, if $\sigma > 0$, then $x(t)$ is a growing exponential; and if $\sigma < 0$, then $x(t)$ is a decaying exponential.




**D. Signum function:**

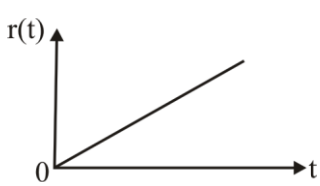



**E. Rectangular pulse or GATE function:**




$$x(t) = A\,rect\left(\frac{t}{\tau}\right) = \begin{cases} A, -\dfrac{\tau}{2} < t < \dfrac{\tau}{2} \\ \dfrac{A}{2}, t = \pm\dfrac{\tau}{2} \\ 0, \text{otherwise} \end{cases}$$

**F. Unit ramp signal:**



$$r(t) = tu(t) = \begin{cases} t,\ t \geq 0 \\ 0,\ otherwise \end{cases}$$

**Example:** Determine the value of $\int_{100}^{\infty} \cos 3t\ \delta(t)\ dt$

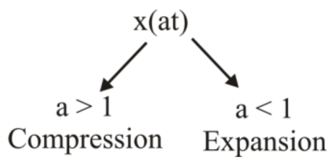
**Solution:** Here $t_0$ lies outside of the limit of the limit of t.

$$So \int_{3}^{\infty} \cos t \delta(t-0) dt = 0$$

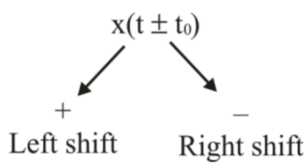
**Problem:** Determine the value of $\int_{2}^{\infty} e^{(t-9)} \delta(t-9) dt$

## J.4 OPERATION ON CONTINUOUS SIGNALS:

**A.** Time scaling

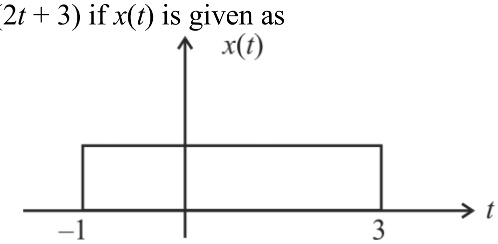



**B.** Time shifting




**C.** Time reversal: x(–t)

**Example**: Draw signal $y(t) = x(2t + 3)$ if $x(t)$ is given as




**Sol.** The problem can be solved in two ways: